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ABSTRACT

This material describes a sequence of inquiry-based instruction with Logo that is designed to teach young children (fourth grade or younger) about pre-proof geometry. Pre-proof geometry includes concepts such as open versus closed paths, angles, lines, polygons, and relations among polygons. Inquiry-based instruction relies upon questions to help a student reflect upon actions. The curriculum is informed by the van Hiele model of thinking in geometry. Logo provides a concrete medium to help children make transitions from visually oriented thought to more descriptively oriented thought. Each instructional session is divided into declarative and procedural interpretations of concepts. Declarative interpretations present the basic facts required to distinguish between instances and non-instances. In contrast, procedural interpretations specify how to create an instance of the concept. The first three lessons are reviews of essential Logo concepts. Fourteen additional sessions for Logo instruction are included. Questions are provided in each session. (YP)

Inquiry-Based Instruction of Pre-proof Geometry with Logo

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Overview of Curriculum

We describe a sequence of inquiry-based instruction with Logo that is designed to teach young children (fourth grade or younger) about pre-proof geometry. Pre-proof geometry includes concepts such as open vs. closed paths, angles, lines, polygons, and relations among polygons. Inquiry-based instruction relies upon questions to help a student reflect upon actions. The curriculum is informed by the van Hiele model of thinking in geometry. Logo provides a concrete medium to help children make transitions from visually oriented thought (e.g. squares are figures that look like boxes) to more descriptively oriented thought (e.g. squares are closed figures with certain properties).

Each instructional session is divided into declarative and procedural interpretations of concepts. Declarative interpretations include the defining features of concepts. They present the basic facts required to distinguish between instances and non-instances. For example, an angle can be defined as the union of two rays. In contrast, procedural interpretations specify how to create an instance of the concept. For example, one can construct a parallelogram with a particular Logo program.

QUESTIONS are included in each session; these were asked to help students elaborate their understanding of a question, to point out inconsistencies, to test how students were thinking about an idea, and the like. Other questions were more precisely tailored to each student's grasp of the situation.

The first three lessons are reviews of essential Logo concepts. We believe that children should be able to comfortably define procedures with at least one variable before embarking on the geometry sequence. The children we taught were all fairly proficient with Logo. More time would be required for children without the same experience with Logo.

Teaching Sessions — Logo Pre-instruction

11/3 Pre-1

Since each student had been exposed to Logo at some time in the past, we decided to use our first three sessions with the students as strictly a recalling of the computer language they had previously learned, and as an introduction into higher order Logo concepts (e.g. procedures with more than one variable).

Our first session was devoted to getting the students back on the computers and also to re-familiarizing them with Logo. Before they started working with the computers, we reviewed some basic Logo concepts with the use of a blackboard. We began by drawing a triangular-shaped object on a blackboard and asked the students for the name of this object when we see it on the computer screen. The answer "turtle" was elicited. We then identified some screen turtle commands that would either hide the turtle from our sight or show the turtle on the screen. These are the hide turtle (HT) and show turtle (ST) commands, respectively. We then proceeded to identify the Logo commands that are used for moves, and those that are moved for turns. These are the forward (FD) and back (BK) commands (for moves), and the right (RT) and left (LT) commands (for turns). These commands also make use of inputs so that the turtle knows how far you want it to move or turn. For example, the command FD 50 tells the turtle that you want it to move forward (the direction the nose is pointing) 50 turtle steps; RT 45 tells the turtle to rotate its nose 45° to its right. Examples of these commands were written on the blackboard and a list was started of the relevant commands that they were learning about. Special attention was paid to the manner in which commands with inputs are written on the screen, i.e., a space must separate the command from its input. The students were questioned about this, and any response which indicated that Logo is fussy about spacing was accepted.

Because of the frequency in which typographical errors occur, the erasure of text was then covered. The students were asked, "Suppose I were typing in LEFT and I accidently typed in LEFF instead. How could I erase the second F?" Answers were elicited and the concept of the delete key was introduced. The clear text (CT) command was also introduced. This command erases all of the text that is present on the screen at the time. But, what about pictures? "How do we erase pictures that are drawn on the screen?" This introduced clear screen (CS). CS does two things. First, it clears the screen of its drawings. Second, it sends the turtle back home, that is, back to the middle of the screen. The commands clean and home were also introduced at this time and related to the CS command.

At times, we want to move the turtle without drawing. We do this by the pen up (PU) command. To return to drawing, we use the pen down (PD) command.

Students then worked in pairs with instructors exploring the effects of the Logo commands in immediate mode. They were asked to predict outcomes of correct and "buggy" examples (e.g. what does FD 50 do?, what about FD50 ?), and to correct the "buggy" examples. The repeat command (with input) was also then reintroduced. The students were reminded that with the repeat command, the turtle "does the same thing again and again" (like with breathing). The students then were asked to take a standard command and make an equivalent using repeat (e.g. FD 100 = Repeat 10 [FD 10]; FD 100 = Repeat 5 [FD 20]). As time permitted, the process of writing procedures was started.

11/5 Pre-2

We began with a very brief review of some of the commands that we covered during the last session and then introduced some screen functions. These screen functions are full screen (FS), split screen (SS), and text screen (TS). The FS command allows only graphics to appear on the screen; the TS command allows only text to appear; and the SS command allows a combination of the other two.

We then went to the computers with the pairs of students and instructor and continued the work begun on the previous day. Some students began with the repeat command, while others began with procedures. There were two types of procedures taught. The first utilized graphics and drawings; the second utilized text. The text procedures were used to introduce the print (PR) command which allows text to be written to the screen. The text procedure was: TO LAUGH, PR [ha! ha! ha!], END and TO CRY, PR [wa! wa! wa!] , END. We then challenged students to make a procedure LAUGH.AND.CRY (This idea is Wally Feurzeig's). In either case, all students did work on the writing and the editing of procedures.

Again, as in the last session, some students were further along than the others. Some students began playing with procedures that played music (e.g. To Toot Tone (duration) End), while others began working on procedures with variables.

11/6 Pre-3

This was our last pre-treatment session. All of the students worked on the utilization of variables within their procedures. Both one and more than one variable procedures were taught. The concept of a variable was described with the analogy of a "box" into which things could be placed. The students were also reminded of things that varied, like the weather. Weather was then equated with the box, and things that could be put into it to change it were cloudy, sunny, rainy, and the like. Some difficulties were observed with the use of variables, however, the majority of the students were competent in variable use and definition at the day's end. We also continued with super-procedures (procedures within procedures) (e.g. To Make.Sounds Toot.1 Toot.2 End).

Teaching Sessions — Logo Instruction

11/10 Session-1

***Geometry Concepts:** form, size, figure, open, closed, simple, non-simple, point, line, line segment, curve*

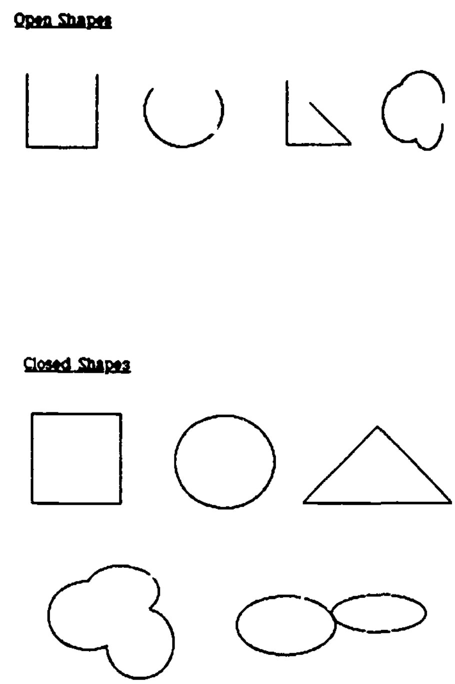
Declarative Specification

We started with an explanation that geometry dealt with form and size. We explained how the form and size of geometric figures always have an effect on our living and thinking from day to day. Everyday, in many ways, we are in contact with both form and size. We then elicited examples in which form and size were parts (e.g. round plate, square box, big apple, large man).

After this brief initiation into geometry, we proceeded to an introduction into open and closed shapes. A closed curve was defined as a curve that ends were it starts; an open curve, a curve that does not end where it starts; a simple closed curve, a curve that begins and ends at the same point and it does not intersect itself. Examples of closed curves, open curves, and simple closed curves were presented. (see Figure S-1a)

We then proceeded to a lesson on points, lines, and line segments. The point was defined as a representation of a position or of a location. It has no size or length. We explained how we usually make a "dot" of some kind to indicate the location of a point. The position of the turtle on the screen at any point in time could be thought of as a point. The line was then defined as a straight path that goes on forever in both directions (in our imaginations). This is difficult to show on the computer because of the size limitation of the screen. However, stress was placed on the concepts of straight, no end, and both directions (using the forward or back commands with a very large number), and the students appeared not to have difficulty with this definition. The line segment was then defined as being a a part of a line (so a part of FD or BK with a "regular" number). It is a straight path with two end points, that is, it has a beginning and an end. This concept is more easily presented on the computer screen. The commands FD and BK (with inputs) move the turtle in a straight path. The end points of the segment, then, are where the turtle begins, and where it ends. Stress was placed on the line segment being straight, that is, it has no RT's or LT's in it, and also its having a beginning and an end. Examples of line segments (FD or BK) and non-examples (FD & RT, or BK & RT, etc.) were presented, and the students were invited to compare the two. QUESTIONS included: Which is longer? How can you be sure? If the turtle starts at different places, are the segments always of different length? How do you know for sure?

S-1a



Procedural Specification

The students were then given the opportunity to construct open and closed curves, and open and closed shapes, with Logo. In the construction of these figures, the majority of the students constructed approximations to circles and to squares for the open figures, and circles and squares (or rectangles) for the closed figures. Most of the students had no difficulty with the open and the closed concept. The concepts were most easily differentiated as, "A closed shape is when the turtle ends at the same place at which it began; open shapes were when the turtle did not return to its original place (and never crosses itself either)." QUESTIONS included: Why is this one open and this one closed? How can you be sure?

An overlay, with six points on it, was also placed on the screen. The students were then challenged to connect the points so that they made line segments.

11/12 Session-2

***Geometry Concepts:** parallel, intersecting lines.*

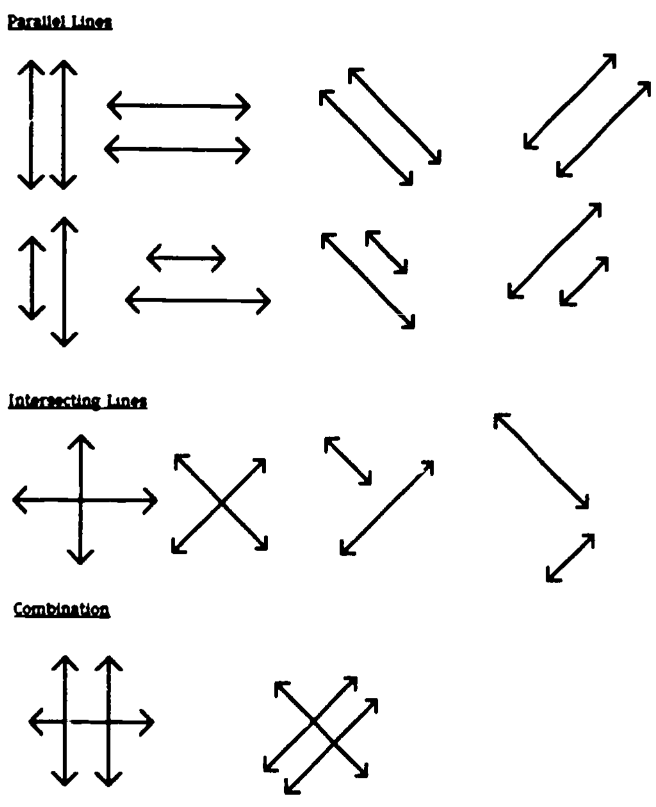
Declarative Specification

We began this session with a review of the material we had covered in the previous session. Open and closed shapes, points, lines, and line segments were reviewed until we felt confident that the students had a good understanding of these concepts. We then proceeded into the area of parallel and intersecting lines. Two straight lines (or segments) that do not cross each other no matter how far they extend was the definition given to parallel lines. As was reviewed previously, the fact that lines continue in each direction forever was reiterated and the pictorial representation of lines and parallel lines was shown. Two straight lines (or segments) that cross each other was the definition given to intersecting lines. Again it was reinforced that lines continue in each direction forever. Another point that was taught in order to help the children further understand the difference between parallel and intersecting lines was that with parallel lines, the distance between the lines stays the same whereas with intersecting lines, the distance changes. The students were then given numerous examples of parallel lines, intersecting lines, and combinations of the two, and comparisons were elicited. (see Figure S-2a) QUESTIONS included those designed to entrap students (based on our previous research) - for example, given a pair on lines that would intersect but did not, we said: "Well, look these two lines never touch, so they must be parallel, right?" Other questions, such as "pretend that I'm on the phone, and tell me again why these lines are parallel (remember, I can't see them)".

Procedural Specification

Some students briefly experimented further with the drawing of open and closed curves and figures. However, all of the students proceeded into constructing parallel and intersecting lines. The students were challenged to make a procedure (using a variable) that would construct a line. For example, TO LS :d, FD :d, END. They were then further challenged to draw sets of parallel and intersecting lines using this procedure. While drawing a line (segment) is a fairly easy task in Logo (using FD and/or BK with inputs), the challenge to make a procedure to do it served several purposes. First, it showed the equivalence of some "primitive" commands (FD, BK) to some more "complex" commands (procedures). Second, it allowed for practice in the construction of procedures using variables. Third, it was a handy tool for measuring the equidistance between parallel lines and the non-equidistance between intersecting lines. Fourth, it facilitated the acquisition of super-procedures. For example, parallel lines could be made by constructing a parallel line procedure which utilized the line procedure, twice, and possibly a move procedure which would pick the turtle's pen up, move the turtle so that it was positioned for parallelism (i.e., it had the same heading or its opposite), and then place the pen down to continue drawing. And lastly, we believed that the use of procedures with variables would aid in the acquisition of the knowledge that line segments do not have to be of equal lengths in order to be parallel.

S-2a



11/13 Session-3

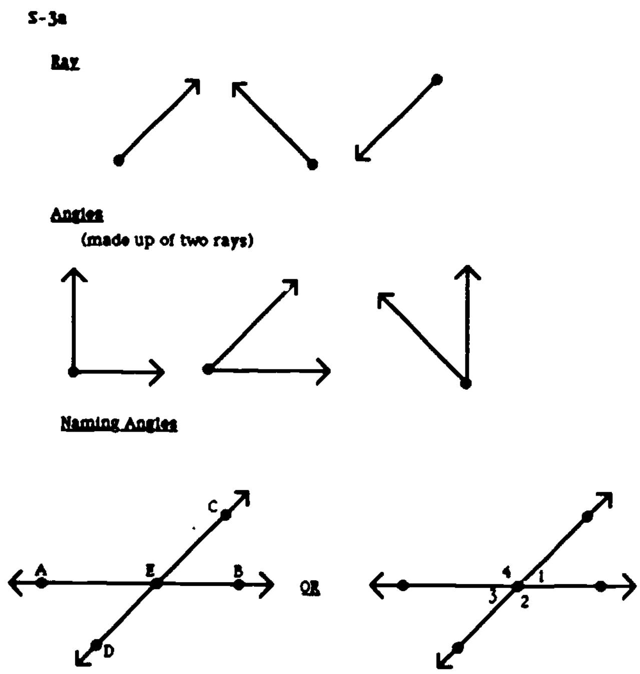
Geometry Concepts: *Rays, angles, vertex, naming angles*

Declarative Specification

Today we began with a follow-up of parallel and intersecting lines. No new material about them was presented, however, any questions that the students had were answered and many of the students continued to work on or with their line procedure(s). We then proceeded into rays and angles. The ray was defined as being a part of a line that has only one end point and continues on forever (in our imaginations) in one direction. We then explained and showed them that two rays that share a common end point form an angle. (see Figure S-3a) This common end point is called the vertex. We introduced a rotational definition of angle by saying that an angle is sometimes thought of as the amount that the turtle turns between two straight lines (or line segments or rays) that meet at a point.

Procedural Specification

We again had the students construct intersecting lines on the computer. (QUESTIONS) We then asked how many angles are formed when two straight lines (and segments) intersect. We elicited the answer, four, and corrected and explained the answer to those who missed it. We also had the students construct angles of different sizes (either with their line procedure or with the "primitive" commands). As a further challenge, we had one student construct an angle while another student estimated how large or how small the angle was. We then (QUESTIONS) questioned the children about how one angle was different from another angle. For example, this one has longer sides so it's bigger, right? How could it be that the measures are the same even when one looks more open here (bigger size on screen but measure of angle the same)? We used this to emphasize the notion that angles differ in the amount of their openings, not in how long or how short their sides are. For example, which angle is bigger? Why? And, given two angles of equal measure but one where the line segments were unequal, we attempted to trap the children into stating that the measures of the angles were different. If successful, we followed up with additional explanation and examples. If not, we asked the children to justify their response to convince us.



A few of the students raised questions concerning the interior and the exterior of the angles. This proved to be a perplexing problem. In the construction of angles, the amount that the turtle turns before the construction of the second side of the angle is the "exterior" of that angle. The actual size of the angle, however, is measured on the "interior" of the angle. This problem brought to the surface the two related 360° theorems. The first is that the sum of the exterior angles of any closed figure made up of line segments is 360°. The second, is the 360° theorem of a circle, that is, a complete circle is made up of 360°. This second theorem was helpful in explaining why a straight line (or straight angle) is 180°. Therefore, in making an angle, when you turn the turtle to make the second line, the interior of the angle must be 180° minus the turn. At this time, this information was only discussed with the few students who were ready to deal with it.

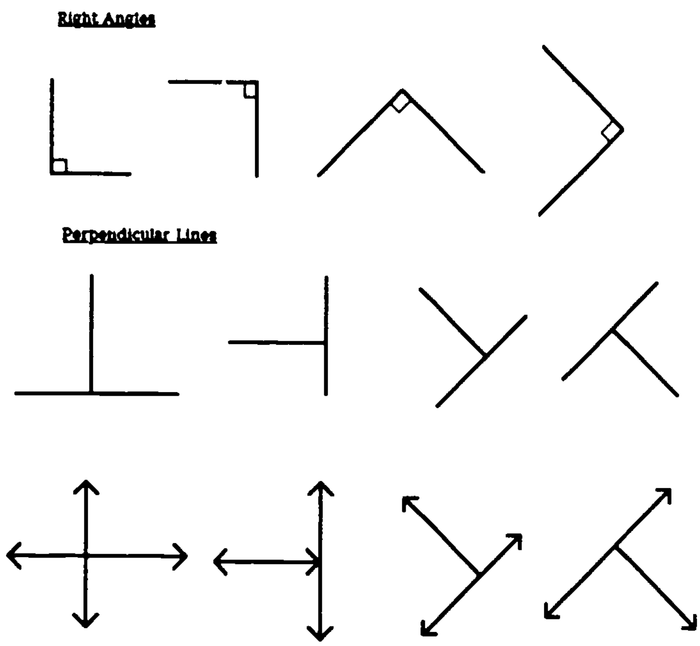
11/17 Session-4

Geometry Concepts: *Identification of right, straight, acute, and obtuse angles.*

Declarative Specification

We began with a brief review of the previous session and initiated the writing of angles. We then moved into the identification and properties of right angles, straight angles, and perpendicular lines. A right angle was defined as an angle that forms a square corner. This is done on the computer by turning RT 90 or LT 90. Many examples of right angles were given, relating these to "real-world" corners in the classroom and elsewhere. Perpendicular lines were then described with intersecting lines that form equal angles next to each other. (see Figure S-4a) QUESTIONS the students were asked included "How many angles are formed in each picture (diagrams included)? Why?" The students were shown that both of the angles were 90° and they were then shown the symbol for perpendicular lines. Examples of perpendicularity in the classroom environment were elicited from the children. The concept of straight angles was now raised for all, and it was defined as a union of rays (emphasizing the vertex at the intersection) and as a turtle rotation of 180°. The 360° theorems were then reviewed.

S-4



Procedural Specification

An angle program was developed as a piece of software for all of the students to use (Angle). This program allowed the students to guess or to estimate ("guess-timate") how large an opening would be formed by specific turtle turning commands in Logo. For example, one line was drawn (random screen orientation), and a child then typed in a turning command (e.g., RT45). The other student then had to "guess-timate" where he/she believed the turtle would draw the second side of the angle.

We also introduced the concept of complementary angles in Logo, that is, LT 90 and RT 270 would both put the turtle in the same place.

11/19 Session-5

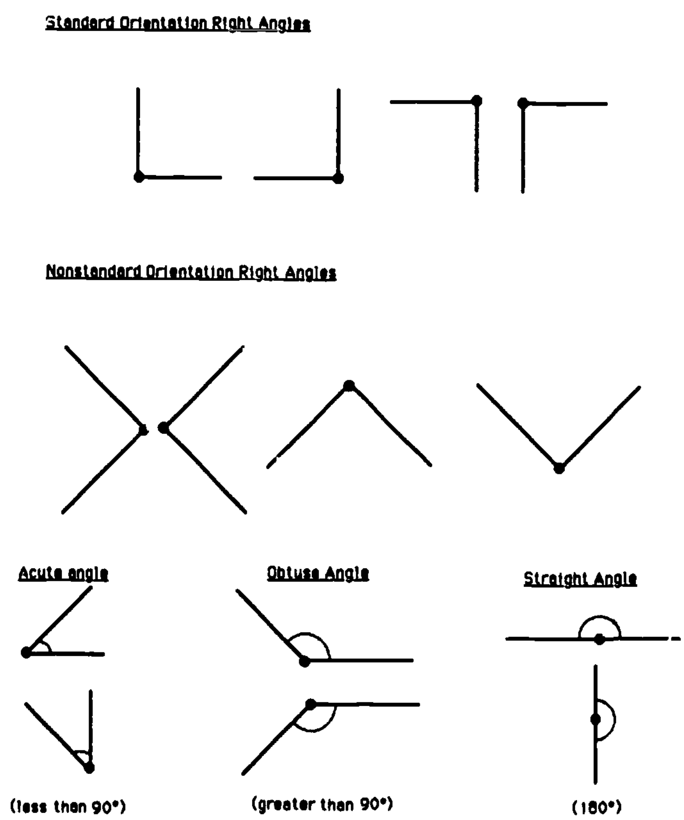
***Geometry Concepts:** angles, identification of acute angles, 360° theorem of a circle, obtuse angles, angle measure.*

Declarative Specification

We began this session with a brief view of the previous session. The students then moved into learning about the different kinds of angles. We showed them that as they had seen before, if two sides of an angle are perpendicular to each other, that is, they form a square corner, the angle formed is called a right angle. With this knowledge of right angles, the students were shown that angles smaller than right angles were called acute angles. Many examples were given and the students had ample opportunity to practice naming these angles. Next in the session we moved into obtuse angles. These were described as any angles greater than a right angle. Again, students were given many examples and had ample opportunity to practice naming these angles. (see Figure S-5a) We did not get into the concept of reflex angles; we left any angle that could be considered reflex as obtuse.

Because the 360° theorems had already been introduced, we reviewed it again with the students.

S-5a



Out of this, the straight angle was again covered (e.g. students described how straight angles were just like any other angles and how they were different).

Procedural Specification

The students were also provided with a second piece of angle software (Guessangle). This software showed the students a randomly drawn angle and asked them to "guess-timate" how large or how small they believed the angle was. It also allowed them to see exactly what size angle they guessed in comparison to the one shown. QUESTIONS concerned how one angle was different from another angle. Emphasis was placed on the notion that angles differ in the amount of their openings, not in how long or how short their sides are. For example, which angle is bigger? Why?

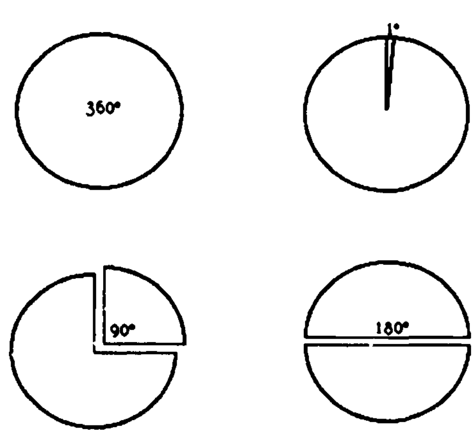
11/20 Session-6

***Geometry Concepts:** Angles, angles as a rotation, estimation of angles*

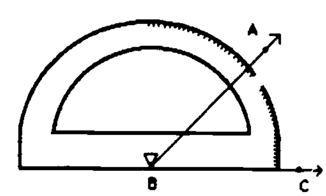
Declarative Specification

This session was a continuation of angles. We began by talking to them about all of the angles that they had previously seen. We then asked them to name all of the angles we had previously covered. The answers right angle, straight angle, acute angle, and obtuse angle were elicited. We then proceeded with a discussion of the Babylonians. We explained how thousands of years ago the Babylonians decided to divide or breakdown a complete circle into 360 parts (they could have decided on any number, but they decided on 360). Each part they called one degree. This unit, 1°, is therefore, 1 part of 360 or 1° out of 360°. One full rotation is then 360°. Since one rotation is 360°, then one right angle is called 90°, and one straight angle is called 180° (pictorial representations shown). (see Figure S-6a)

S-6a



Place a protractor so that the arrow is on vertex B and the edge is on Ray BC



Read the measure of the angle where Ray BA crosses the scale

Procedural Specification

The students were again provided with both pieces of "angle" software. They used the first piece of "angle" software to construct acute, obtuse, right, and straight angles. They also used the other piece (GuessAngle) to continue "guess-timating" at the sizes of angles. In conjunction with this software, we used a "prove" activity (with appropriate software) which enabled them to construct angles and see that the angles constructed were indeed portions of a circle just like the Babylonians said. The "prove" activity began with the students constructing a line (a FD & BK the same number), a turn (RT or LT), and the line again. The turtle was then brought to the end point of one of the line segments, and an arc was drawn "exactly" between the two line segments. For example, if the turn was an RT 45, Repeat 45 [FD 1 RT 1] was used to draw the arc. By varying the perimeter of the circle (and hence the length of the line segments), we attempted to get the notion across that the rays may go on and on, but the "opening" measured is the portion of the circle intersected by the rays, nothing more.

11/24 Session-7

***Geometry Concepts:** Angle measure, angle estimation*

Declarative Specification

This session was used as a review of angles and also of variables. We briefly recovered the angle material of the last sessions, and also the 360° concept as it related to angles. We then reviewed the use of variables so that the students could use them in making procedures for the "prove" activity.

Procedural Specification

The students continued to make use of the "angle" and "GuessAngle" programs and overall, they exhibited excellent angle size estimation. We then proceeded further into the construction of angles, using variables whenever possible (e.g., in the construction of line segments). Some students still had difficulties with the variable concept, and these students were further instructed in their pairs.

11/26 Session-8

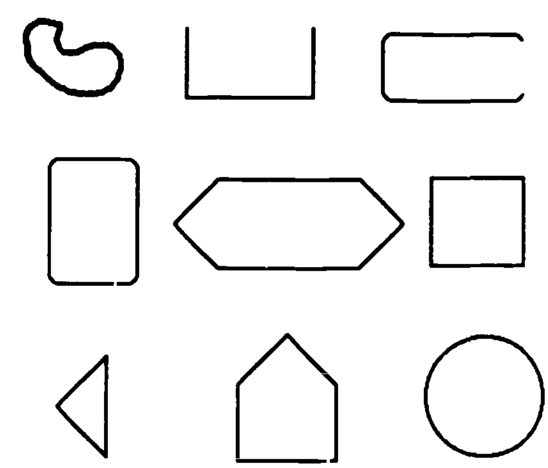
***Geometry Concepts:** Polygons, pentagon, hexagon, triangles, perimeter.*

Declarative Specification

We started this session with a very quick review of some past material. Lines, line segments, points, rays, vertices, and open and closed curves are just a few of the reviewed materials covered. We then began our introduction into polygons. A polygon was defined as a simple closed curve composed of line segments. This definition was tied into information that was previously learned and subsequently reviewed. The word polygon was also broken down into parts (poly-> many, gon-> angles) and the idea of a many angled figure was presented. We then related the Pentagon in Washington, D.C. to our discussion. This five-sided, five-angled building was used to introduce the many angled figures and also to introduce the Greek and Latin prefixes used in naming the figures (e.g. penta->five...deca->ten).

The students were told that if the figure is not closed, then it is not a polygon. They were also told that the figure must be composed of line segments. Examples and non-examples of polygons were drawn on a blackboard and answers as to a figures "polygon-ness" were elicited. (see Figure S-8a) Both correct and incorrect answers were elaborated on until we felt the students understood the concept. These elaborations included identification of hexagons, pentagons, decagons, and the like. QUESTIONS included the by now usual attempts to entrap students. For example, is this one (a closed figure with 4 line segments and a curved segment) a polygon? Why? or why not? The line segments that form the boundary of a polygon were called the sides of the polygon and the perimeter of the polygon was described as the sum of the sides or the distance around the polygon. Again the term vertex was introduced but this time, it was described as the point at which the sides of a polygon meet (this was tied into their previous definition of vertex, i.e. the point were two lines, line segments, or rays meet).

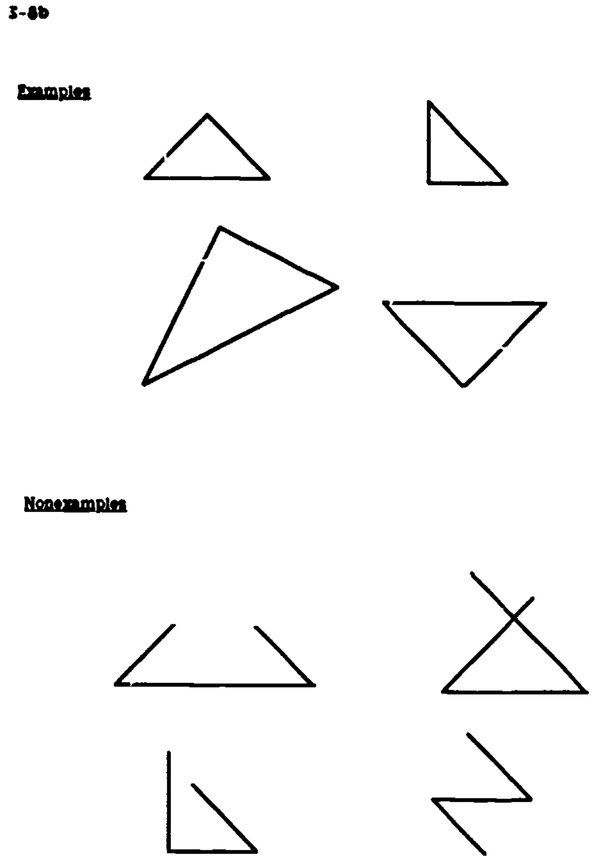
S-8a



We then began our introduction into triangles. Three non-colinear points were placed on the blackboard and labelled A, B, & C, respectively. We then connected the points with line segments. The question was then posed as to whether or not this figure was a simple closed curve. Answers were elicited, and questions were asked to look again at students' reasoning. They were then told that this figure was called a triangle. Answers were elicited from the students as to the figure's number of sides, angles, and vertices. They were also given a review and then practice in naming the sides and angles of the polygon just as they had previously done with line segments and angles. The triangle was then defined as any simple, closed shape composed of three line segments, three angles, and three vertices. QUESTIONS included: "What will happen if one point goes right on top of another point? Will it still be a triangle? " They were also introduced to the right triangle, that is, a triangle with one right angle. They were reminded about the 360° theorem, and were given help in determining how to construct an equilateral triangle (e.g., Repeat 3 [RT 120 FD 50]. Examples and non-examples of triangles (see Figure S-8b) were constructed, and reasons why or why not a certain figure was or wasn't a triangle were discussed (e.g., closed, 3 sides, & 3 angles = triangle). We especially cautioned children that "looks like" did not a triangle make.

Procedural Specification

During this session the students were placed at the computers and asked to construct triangles. The easiest triangle for them to draw was the equilateral triangle. This triangle has all of its' sides and angles (and turns) the same. They were challenged to draw these triangles with and without the use of the repeat command. They were also challenged to construct various other triangles (e.g., right, isosceles, etc.). This proved to be a very difficult task for the children. Coordinating the length of the third side and the amount of turn needed to complete the 360° appeared to be too heavy a load. (In the future we would advise provision of some psuedo-primitives to make this task easier).



12/1 Session-9

Geometry Concepts: *Polygons, triangles, quadrilaterals, rectangle, square*

Declarative Specification

We had covered a lot of information, new and old, in our last session with the kids and because of a few school days off, we began with a brief review of polygons and triangles before moving to new material. After a reacquaintance with these terms and figures, we proceeded to teach the students about quadrilaterals, that is, a polygon (with all of its meanings) with four sides (its four angles and four vertices were also emphasized). The progression of QUESTIONS went like this: Is this (a four-sided convex quadrilateral was placed on the board) a polygon? How do you know (closed, line segments, angles)? Is this figure also a quadrilateral? How do you know? So can we say that a quadrilateral is a kind of polygon? Why or why not?

A special kind of quadrilateral was then introduced. This special quadrilateral was the rectangle. We could have started more generally with a parallelogram, but we felt that the children were much more familiar with the rectangle. A rectangle was drawn on the blackboard. Questions were: Is this a quadrilateral? Why, How do you know? Is it a polygon? How many angles does it have? After answers to these questions were received, we asked the students if they noticed anything special about the angles of the rectangle. The rectangle is a special quadrilateral with four sides and four right angles (i.e. 4 90 turtle turns). A list of properties was then started on the blackboard. We then proceeded to label the vertices of the rectangle and introduced the students to the concept of opposite sides of the figure. We measured all of the sides of the rectangle, and then we asked them what they noticed. After receiving their answers, the fact that the opposite sides of a rectangle have the same length and are parallel was then added to their definition and to the property list which already consisted of closed polygon, four sides, and four right angles.

We then briefly moved to a second special kind of quadrilateral. This special quadrilateral was the square. A square was drawn on the blackboard and its defining features were elicited and listed on the blackboard, comparing these with those of the rectangle. Question: Is this a kind of rectangle? Why or why not? So students wound up with side-by-side lists of the properties of a rectangle and of a square. This allowed students to see that a square had every property of a rectangle and one more additional property. This helped anchor class inclusion to numberness of properties.

Procedural Specification

First, children constructed a rectangle in immediate mode without constructing a procedure. (For example, FD 50, RT 90, FD 80, RT 90, FD 50, RT 90, FD 80, RT 90) They then constructed a rectangle procedure, without any variables. After successful execution and debugging of this procedure, we used the editor to re-display the procedure to highlight the repetition of commands (e.g. FD 50 RT 90 FD 20 RT 90 followed by an identical set of commands). Children executed this revised procedure, in which nothing had been changed other than the visual arrangement of the commands. We then had the children go back to the editor and re-write the rectangle procedure with the Repeat command, followed by the two lines of the re-arranged rectangle procedure. That is, TO RECTANGLE, repeat 2 [FD 50 RT 90 FD 20 RT 90] followed by FD 50 RT 90 FD 20 RT 90 , FD 50 RT 90 FD 20 RT 90, End. QUESTIONS concerned the relationship between the commands and the properties of the figures observed. Also, "Why does the first line of the procedure do the same thing as the next two lines?" We also included a SLOW procedure that made the turtle wait between the

execution of each line to help the child see the turtle's action more clearly. Other QUESTIONS included: How can you be sure that the opposite sides are parallel? If the opposite sides are equal and the angles are 90, must the lines be parallel?
It is important to slow the action of the turtle down!

12/3 Session-10

Geometry Concepts: *Inclusion relationship between a square and a rectangle.*

Declarative Specification

We continued in this session the discussion of squares and rectangles.
Both figures were drawn of the blackboard and the question as to whether the square was a rectangle or not was posed to the class. The question generated many interesting responses. The gist of the responses was that a square was a square, not a rectangle. We went step-by-step through the definition and the property list of a rectangle and of a square, at each feature testing the figures. We then concluded that a square was a special kind of rectangle because the square has all of the properties of a rectangle; what makes the the square a special rectangle is that it has one more property, that is, all four of its sides are the same length. The effect of this was either an "Oh !" or a very confused look but, in either case, the majority of the students did not grasp immediately this means for defining the class inclusion concept. Although some students did in fact appear to grasp the concept, however, some further believed that because a square is a rectangle, then a rectangle is a square. We confirmed this with an entrapment question: " Then every rectangle must be a square, right? "We countered this belief by presenting several rectangles that were not squares and then asked the children if the drawn rectangles had all of the properties of the square. We concluded that the rectangle had almost all of the properties that the square had, but, almost only counts in horseshoes!

Procedural Specification

Children edited their rectangle procedure to make the length of one of the sides variable: children then generated examples of rectangles. The procedure was then edited to include a variable for the length .of each side. QUESTION: " How can you use your rectangle procedure to make a square?" What must be true? Is every square a rectangle? Is every rectangle a square? Why or why not? How is the square like the rectangle? How is it different? What did you do that was the same? different? Use RECTANGLE to find out. QUESTION: "I can use RECTANGLE 39 40 to make this figure. Is it a square? It sure looks like one, doesn't it?"

12/4 Session-11

We used this day as a catch-up day. Some were having trouble with the two-variable form of the rectangle procedure, others were simply taking longer to type in the Logo commands than we had anticipated. We briefly reviewed properties of a square and of a rectangle with children, and the use of the SLOW procedure to make the action of the turtle more visible. Children then all used the two-variable form of the rectangle procedure to catch-up to the curriculum (so to speak). Those pairs that were current used Logo to create other polygons (e.g. like they had in previous years) or just "had fun" with Logo.

12/8 Session-12

Geometry Concepts: *Parallel lines, parallelogram, trapezoid*

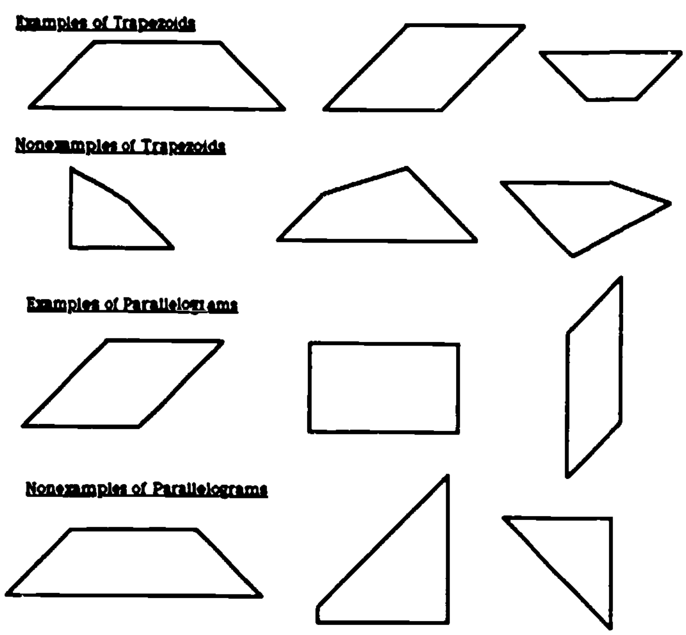
Declarative Specification

We began with a review of parallel lines and line segments. We reviewed this material because we were now planning to cover another type of special quadrilateral, the parallelogram. We began by telling the students that there are special quadrilaterals formed with parallel lines (we asked if they could recall any quads with parallel lines - e.g. the square and the rectangle). We then drew two nonequivalent, parallel line segments on the blackboard. We connected the end points and asked how many sides the figure had. After receiving answers, we asked what could be said about a pair of sides (pointing to the parallel line segments). After eliciting responses to this question, we explained that a trapezoid is a quadrilateral with at least one pair of sides parallel. Examples and non-examples of trapezoids were generated. (see Figure S-12a)

We then drew two pairs of parallel line segments on the blackboard and asked what could be said of these sides. We explained that a quadrilateral with two pairs of sides parallel is called a parallelogram. The property of "the opposite angles are equal" was also included for all parallelograms. A related property for the parallelograms was that the sum of the adjacent angles was 180.

We again encountered difficulty with the class inclusion concept. In every instance, a parallelogram (2 pr. of sides parallel) is a trapezoid (at least 1 pr. of sides parallel). However, not all trapezoids are parallelograms. (see Figure S-12a). We did not dwell on this inclusion relationship, in part because some texts present the relationship as a logical OR (trapezoid or parallelogram).

S-12a



Procedural Specification

We provided children with a procedure that drew a parallelogram, called PARA1. (TO PARA1, FD 60 RT 80 FD 75 RT 100, slow, FD 60 RT 80 FD 75 RT 100, End) QUESTIONS: How long is each side? How much is each angle? What do you notice about the opposite angles? Did you notice that we used the same set of commands two times? What can we do to rewrite this procedure?

We provided children with a second procedure, PARA2, that simply changed the inputs. QUESTIONS were aimed at getting them to discover differences between the two procedures and corresponding differences between the two figures. What did you notice about the opposite sides? (equal and parallel) What makes them that way in the procedure? What did you notice about the turtle turns? What do they always add up to? Why?

We then asked children to write a more general parallelogram procedure with variables for the angles (We assisted them in this). QUESTIONS: Try to make as many parallelograms as you can. Try to make some that are not parallelograms - How can you get the figure to be open? (so the sides don't meet) (Elicit the notion that the adjacent angle sum must be 180 - and that the sum of the angles must be 360).

12/10 Session-13

Geometry Concepts: *Class inclusion hierarchy for quads.*

Declarative Specification

We continued in our teaching of special quadrilaterals and we attempted to clarify some of the difficulties that the students were having with the material presented previously. We then reintroduced the square and the rectangle, focusing this time on the property of parallelism. A property list was again constructed, but this time for parallelograms, rectangles, and squares. The rectangle and the square were now both parallelograms (in addition to quadrilaterals, and the like) with four right angles. QUESTIONS concerned class inclusion relations, the reasons thereof, and so on. For example, Why is every rectangle a parallelogram? Why is every parallelogram not also a rectangle? The square is also a rectangle, which is also a parallelogram, which is also a trapezoid, which is also a quadrilateral with all its sides the same length. Why?

The rhombus was also introduced as a parallelogram with all four of its sides the same length (its angles may or may not be all 90°). QUESTION: is a square a rhombus? Is the square the only quad with all sides of the same length? Is every rhombus a square? How do you know? Let's use the property lists to find out.... Thus, the square is also a rhombus, but a rhombus is not necessarily a square.

Procedural Specification

Try making a rectangle with PARA. QUESTIONS What do you have to make each angle? Compare PARA 90 90 with RECTANGLE 50 80. What's the same? What's different? How can you change PARA do it can draw a rectangle of any size?

To make a rectangle of any size, help the child construct a PARA procedure with 4 variables (the two angles are already in place). Use the REPEAT command to simplify the syntax. Insert SLOW to help make the action of the turtle more visible to children. Have children create instances of rectangles, including squares, with the 4-variable PARA procedure. Time permitting, have children construct a rhombus with PARA. QUESTIONS Is a square the only kind of parallelogram with all sides equal?

12/11 Session-14

Declarative Specification

Today was the last teaching session with the students. We spent much of the time continuing on special quadrilaterals. We stressed the relations among parallelograms (especially the new one about the rhombus and the square). QUESTIONS were again much like those of the previous sessions, prompting children to reflect on the similarities and differences among various quads, both at the level of property declaration and at the level of construction steps required. For example, if I know that the opposite sides of a quadrilateral are parallel, what else do I know? If a figure is both a rhombus and a rectangle, what is it?

Procedural Specification

Use PARA to make a parallelogram that is not a rhombus or a square or a rectangle. Use PARA to find out how a rhombus and a square are the same. Different?